The Age of Things: Sticks Stones and the Universe
Tenth Lecture Notes
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# Parameterizing the Age of the Universe

As we discussed in the last lecture, the redshift and distance data from various galaxies indicate that the universe has been expanding (that is, the average distance between galaxies has been increasing with time). Extrapolating this trend backward in time, it appears that around 15 billion years ago, the distances between galaxies were effectively zero, and all of these objects would have been packed into a very small volume with virtually infinite density. This singular situation marks a critical, formative moment in the history of the universe, which is called the Big Bang. To better determine when this event actually occurred, we must take into account the effects of the composition and geometry of the universe. Fortunately, recent observations have now provided good measurements of these quantities.

## 1 A Closer Look at the Expanding Universe

First, let us take a closer look at the expansion history of the universe documented in the galaxy data. Recall from the previous lecture that in general relativity, gravity is a distortion in the geometry of space and time caused by the presence of matter and energy. These distortions can induce curvature into space and time, and thus cause objects to follow curved paths. However, changing the geometry of space can also alter the distances between objects and produce an expanding universe. These changes in the distances between objects (like galaxies) are quantified with a parameter called the **scale factor**, which is a relative measure of the "size" (or better, scale) of the universe. The scale factor is conventionally set to a value of 1 today. If at some time in the past, the scale factor was 0.5, then the distance between any pair of galaxies at that time was one half the distance between the same pair of galaxies measured today.

The redshift of light from a galaxy can be used to deduce the scale factor of the universe when the light was emitted from the galaxy. The time that has elapsed between when the light was emitted and when it was received here on earth can also be computed if the distance to the galaxy can be determined (given some additional information about the geometry of the universe, see below). Over the last few years, observations of supernovae have provided distances and redshifts of over a hundred distant galaxies. These data have yielded detailed information about how the scale factor has changed over the last 10 billion years, illustrated in figure 1. Clearly, the further away the galaxy is (and the longer it took for its light to reach earth) the smaller the scale factor was when the light was generated. The scale factor has therefore been increasing with time, and was smaller in the past.

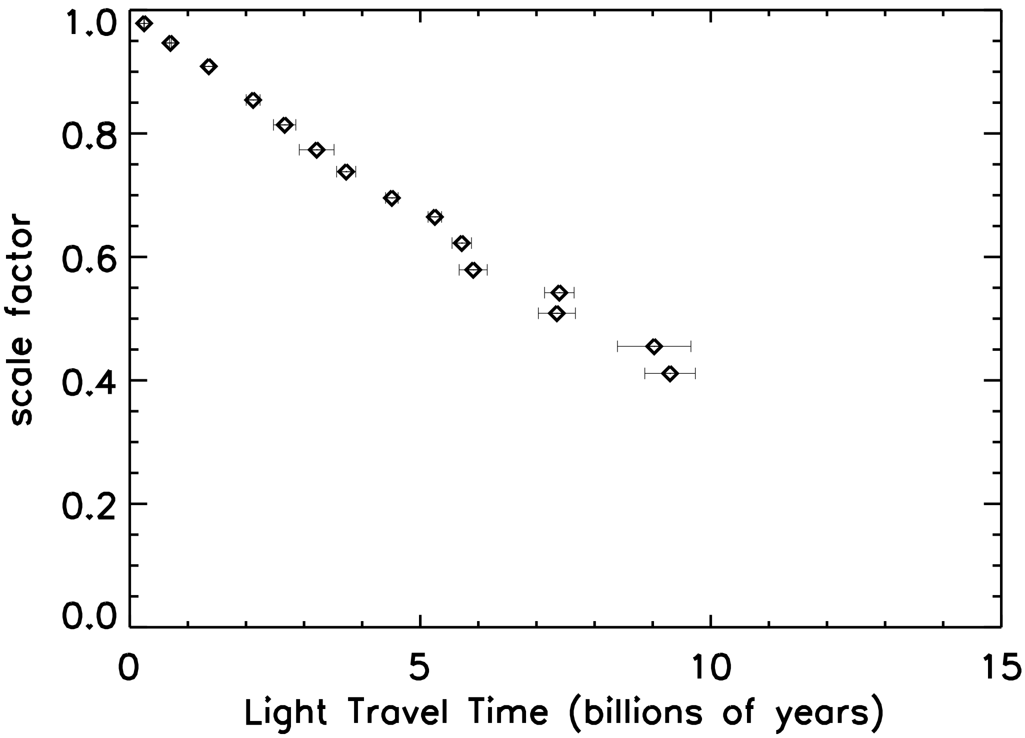



Figure 1: The scale factor of the universe versus time. This graph shows estimates of the scale factor (a measure of the average distance between galaxies, normalized to one today) at various times in the past deduced from supernova data from Riess et. al. (available at `http://www.arxiv.org/abs/astro-ph/0402512` ). Note that in this plot the universe is assumed to have a euclidean spatial geometry. A similar graph was given in the previous lecture notes, which had a separate point for each individual galaxy. For clarity, in this version of the plot the data from all galaxies within a certain range of scale factors are combined to produce each of the points.

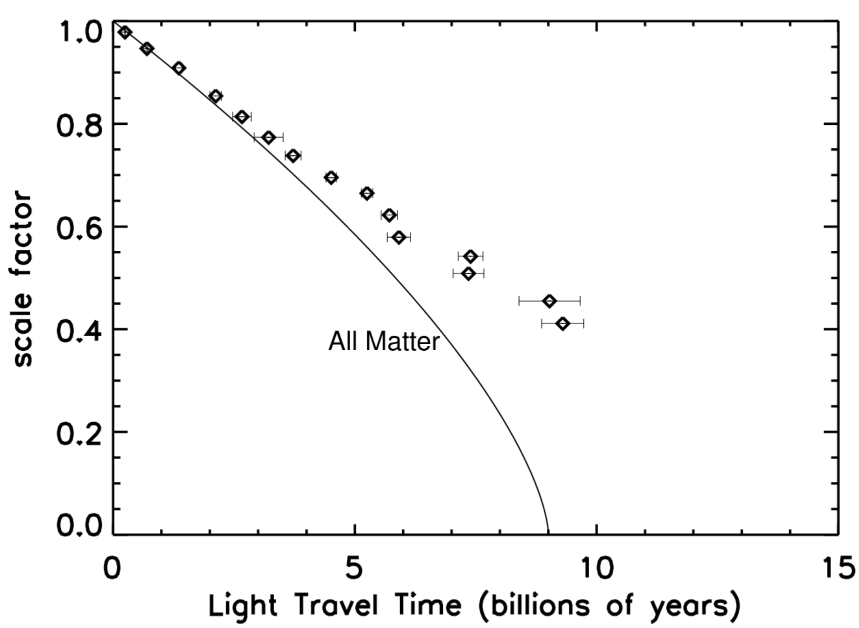
### 1.1 Finding the Recipe for the Universe

The singular conditions of the Big Bang correspond to a universe with a scale factor of essentially zero. In order to estimate when this condition existed, we must extrapolate beyond the available galaxy data. Since parameters which describe the geometry of space and time (like the scale factor) are dynamically coupled to the amount of matter and energy in the universe, a reliable calculation of the timing of the Big Bang requires some information about the material content of the universe. While many characteristics of much of the material in the universe are still mysterious, the qualities which are most relevant to estimating the age of the universe can be reasonably well determined based on existing observations.

In particular, the most important parameter for calculating the age of the universe is the mean **energy density** of the universe (that is, the average amount of energy contained in each unit of volume). This parameter determines the **expansion rate**, or the rate at which the scale factor changes with time: the higher the energy density, the faster the expansion rate. However, as the scale factor changes, the average distance between particles changes, which in general alters the energy density of the universe (and the expansion rate). Therefore to determine exactly how the scale factor should evolve as a function of time, we need to know both the energy density of the universe and how this energy density changes with the scale factor (the latter being known as the material's **equation of state**).

A careful analysis of the data in figure 1 provides important constraints on the material content of the universe, particularly the relative amounts of different materials with different equations of state. For example, it provides a measure of the relative amounts of matter and Dark Energy in the universe. **Matter** in cosmology is a general term, which refers to any material where the majority of the energy is contained in the mass of particles (this is in contrast to **radiation**, which is material with a significant fraction of its total energy contained in the motion of particles). Ordinary objects like atoms are one form of matter, but so is the infamous and mysterious **Dark Matter**. For both these materials, the energy density is proportional to the density of particles, which is very sensitive to the scale factor. Imagine the scale factor increases from 0.5 to 1, then the average distance between particles increases by a factor of two, and the energy density *drops* by a factor of eight. The energy density in matter therefore declines rapidly as the universe expands. This means that the expansion rate gets progressively slower as the scale factor gets larger. (This makes sense, since gravity should pull massive objects together, so the particles of matter should "slow down" as they get further apart.)

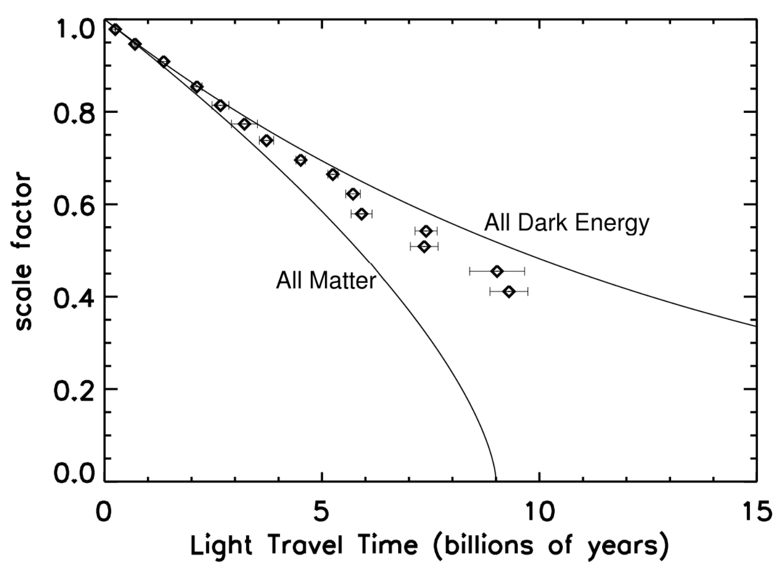
Say we assume that most of the energy in the universe is in the form of matter, then we can calculate how the scale factor should change with time. Since the universe expands more quickly when the scale factor was smaller and the density of particles was higher, we expect the slope of the curve will get progressively steeper further back in time. Such a theoretical curve is shown here, along with the observed data from figure 1.



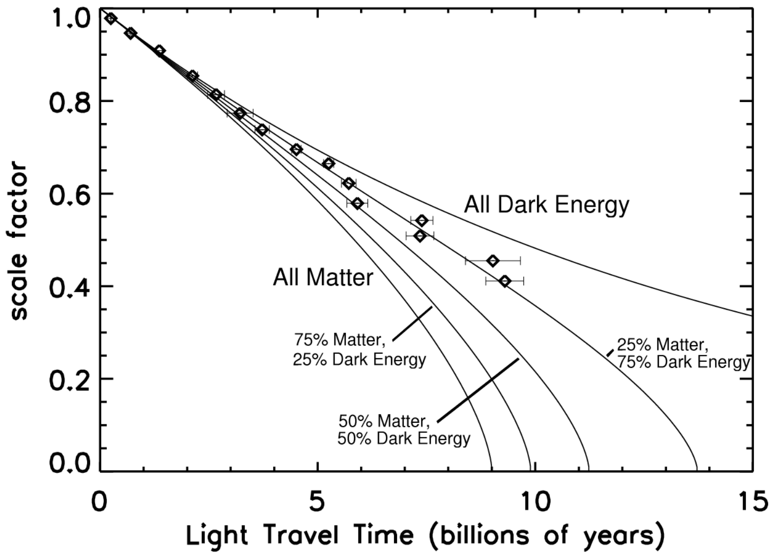

The data clearly do not follow the predicted curve. This result has caused a lot of excitement in recent years. However, it was also not a complete surprise. It had long been recognized that there could be other forms of energy in the universe besides that due to the mass in matter. These data were the first direct evidence that such forms of **Dark Energy** could actually exist.

The most elementary form of Dark Energy is **vacuum energy** (also called the **Cosmological Constant**), which is an energy associated with space itself. If there is vacuum energy, then there is an energy per unit volume of space even it there is nothing around. Since the amount of vacuum energy stored in a volume is unrelated to the number of particles in the volume, the density of this energy is independent of the scale factor. Therefore if most of the energy in the universe is in the form of vacuum energy, then the expansion rate will not decrease as the scale factor increases. In fact, since the energy density in vacuum energy is a constant in time, the ratio of the expansion rate to the scale factor is also a constant, and the expansion rate gets faster and faster as the scale factor increases.

There are other conceivable forms of dark energy besides vacuum energy, for which the energy density does not remain exactly constant with changes in scale factor. However, vacuum energy is still a viable option and for simplicity we will assume here that the Dark Energy behaves like vacuum energy. If all the energy in the universe were in the form of Dark Energy, then it is again straightforward to compute exactly how the scale factor should evolve with time. In this case the slope of the curve becomes progressively shallower in the past as the scale factor decreases. Such a curve is shown in the plot below, along with the data and the matter-only theoretical curve for reference:



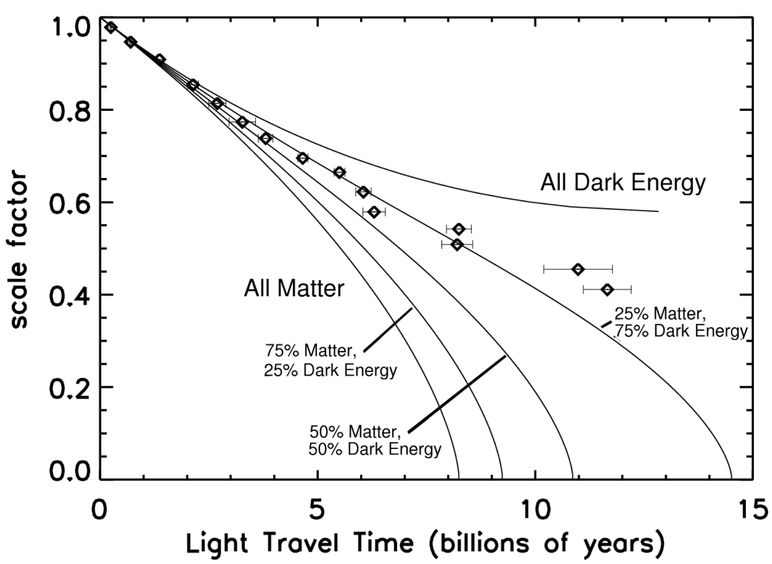

While the data is closer to this curve than the "All Matter" curve, the fit is still not particularly good. In fact, the data falls between the "All Matter" and the "All Dark Energy" curves, so the universe probably contains a mix of dark energy and matter. Note that the energy density of matter declines as the universe expands, while the energy density of Dark Energy stays constant, so the proportion of the energy density in each of these two forms will change with time. This complicates the calculation a bit, but we can still create well-defined theoretical curves for different mixes of matter and Dark Energy, as shown below:




Note that the labels refer to the *current* proportions of matter and dark energy in the universe. The data most closely follow the curve with 75% of the energy density today in the form of Dark Energy. Therefore, these data indicate the universe today contains about three parts dark energy for every one part matter.

## 1.2 Looking Critically at the Total Density

To determine the age of the universe, we need to know not only the relative amounts of matter and Dark Energy, but also the total energy density of the universe. In all the previous plots the total energy density today was assumed to be a certain value called the critical density (see below). If we instead say the total energy density of the universe today was 50% higher than this, the plot would look like this:




The theoretical curves are somewhat different from those in the previous plot, which makes sense because changing the energy density changes the expansion rate.

However, the data points have also shifted to the left. This occurs because the total energy density of the universe can alter the geometry of space and time and affect the apparent distances to objects (we will see a good example of this in the next section). The amount of time it takes for the light from these objects to reach earth is calculated based on these apparent distances and is therefore also influenced by the total energy density of the universe.

While in this plot the total energy density of the universe today is 50% higher than it was in the previous plot, the data still favor the model with about 75% of the current energy density in Dark Energy and 25% in matter. These data thus constrain the mix of Dark Energy and matter regardless of the total energy density. However, since the data match a curve reasonably well in both of the above plots, these observations do not provide a strong constraint on the total energy density of the universe. Thus we cannot derive a robust estimate of the age of the universe from these data alone.

Look at the last two plots and see where the appropriate curves intersect the x-axis. This point gives the time when the theory predicts the scale factor would be zero and the big bang would have occurred. In the earlier plot (which assumes the energy density of the universe equals critical density), the big bang should have happened less than 14 billion years ago. In the later plot (with the higher energy density) this event would have happened more than 14 billion years ago. Therefore, we cannot obtain a precise estimate of the age of the universe without some measure of the total energy density of the universe. Fortunately, recent observations of the Cosmic Microwave Background have placed very tight constraints on this parameter.

## 2 The Cosmic Microwave Background

### 2.1 What is the Cosmic Microwave Background?

As its name implies, the Cosmic Microwave Background, or **CMB**, consists of microwave radiation from outer space. **Microwaves** are a type of electromagnetic radiation (like visible light) with wavelengths around a centimeter to a millimeter. These wavelengths are shorter than the wavelengths of classic radio waves (like those used in television or radio broadcasts), and are longer than the wavelengths of infrared light.

Telescopes sensitive to microwave radiation detect a signal from outer space from every single point on the sky. This **background** of microwave radiation is (almost) constant across the entire sky, and appears to fill all of space. It also has a distinctive spectrum which is shown in figure 2. This spectrum has a broad peak, much like the thermal emission from stars discussed in the eighth lecture. Indeed, the shape of the spectrum corresponds perfectly (within experimental uncertainties) with the theoretical spectrum of thermal radiation from a **blackbody** (an object that absorbs all the light incident upon it). Since the peak of the spectrum occurs at such long wavelengths, the corresponding effective temperature is very low, only a few degrees above absolute zero.

The ubiquity and the spectrum of this microwave background indicates that it is a relic from the early universe (i.e. it is cosmological). In order for the CMB to have such a perfect blackbody spectrum, it must have been produced by material that

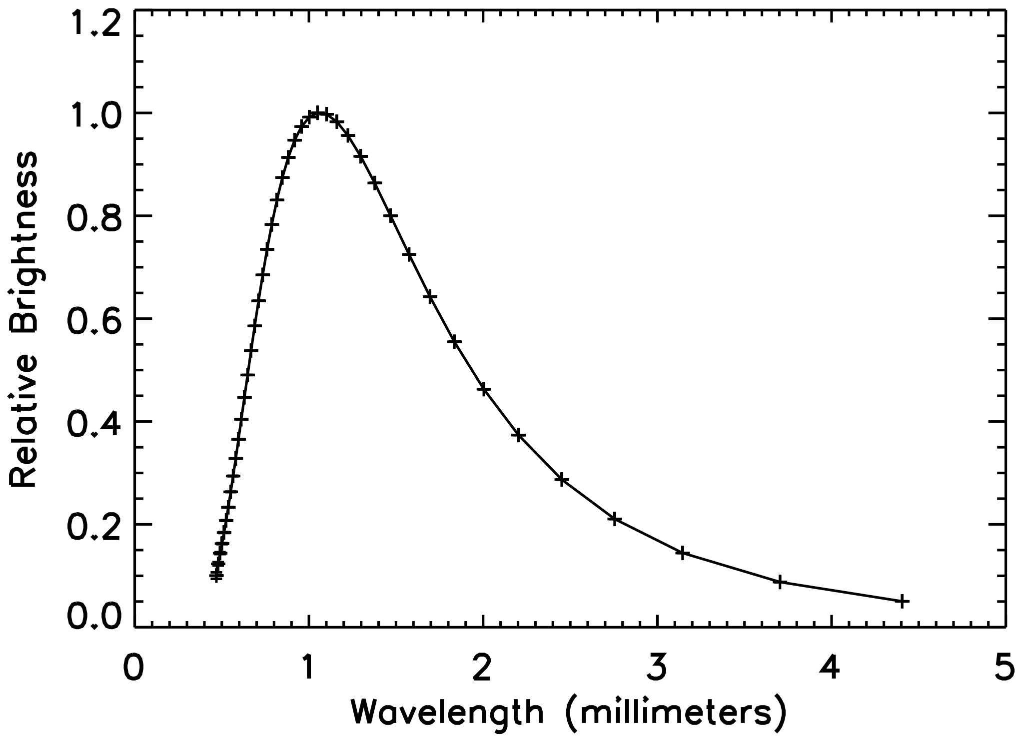



Figure 2: The spectrum of the Cosmic Microwave Background. The points show the observed relative brightness of the CMB as a function of wavelength (from Fixsen et al "The Cosmic Microwave Background Spectrum from the Full *COBE* FIRAS Data Set" in *The Astrophysical Journal* Volume 473 (1996) page 573-586). The curve is the theoretical spectrum of the thermal radiation from a 2.7 Kelvin blackbody. The theory and observations match extremely well.

interacts strongly with electromagnetic radiation over a broad range of wavelengths. Free subatomic particles, like electrons and nuclei couple strongly to electromagnetic waves because they have a net electric charge. However, at low temperatures such particles are usually found together in atoms, which have no net electric charge and only absorb or emit light at certain special wavelengths. Indeed, blackbody radiation is typically produced only in material which is so hot it that acts as a **plasma** (a collection of free electrons and nuclei). Metals and other solids can produce blackbody radiation at lower temperatures, but the dust and hydrogen gas in the universe almost certainly cannot generate such a perfect blackbody spectrum at only a few degrees above absolute zero.

Both the blackbody shape of the spectrum and its extremely low effective temperature can be explained in the context of an expanding universe. As the universe expands, the same amount of radiation is spread over a larger volume. Furthermore, the wavelength of every single photon gets stretched. The spectrum of the CMB has therefore changed as the universe has expanded. It turns out that as the universe expands, radiation with a blackbody spectrum spectrum retains its characteristic shape, but the apparent temperature decreases. Indeed, if the scale factor of the universe doubles, the apparent temperature of the CMB gets cut in half. Therefore, in the past, when the scale factor was smaller, the temperature of the CMB was higher (there is actually evidence for this from observations of the microwave radiation in the vicinity of clusters of galaxies) and at sufficiently early times, the effective temperature of the CMB would be consistent with the temperature of a plasma.

The CMB therefore originates from a hot, dense phase of the early universe. Shortly after the big bang, the material was extremely hot, and the universe was filled with a plasma consisting of free electrons and nuclei and high-energy radiation like X-rays and ultraviolet light. If an electron and a nuclei got together and formed an atom of neutral hydrogen, it would not take long before a high-energy photon would come along and break the atom back into its component parts. As the universe expanded, it cooled as the photons became more spread out and the wavelengths of the photons became longer and longer. Eventually, about 400,000 years after the Big Bang, there was simply not enough ultraviolet radiation around to keep the universe ionized. At this time, electrons and nuclei combined to form neutral atoms and the universe came to be filled with transparent hydrogen gas. This time in the history of the universe is known as **decoupling**, because the photons are no longer strongly coupled to the atoms in the universe. Since the photons no longer interact strongly with the matter in the universe, they travel in "straight" lines (where the definition of "straight" depends on the large-scale geometry of the universe). Indeed, this radiation has been traveling on relatively straight lines ever since the universe was 400,000 years old. During this time, the photons have redshifted by a factor of 1000 from visible and ultraviolet wavelengths all the way into the microwave range.

## 2.2 Variations in the CMB and the Geometry of the Universe

Since the photons that make up the CMB have traveled in roughly straight lines between decoupling and today, the radiation that comes to us from different directions in the sky comes from different regions of the early universe. Therefore any variations in the characteristics of the CMB from point to point across the sky correspond to variations in the structure of the early universe. In particular, there are small (about one part in 10,000) variations in the brightness of the CMB which have now been measured with exquisite precision by the WMAP satellite (see figure 3). The places where the CMB appears somewhat brighter than average correspond to regions of the universe which were slighter warmer and denser than average, while places where the CMB is a little dimmer corresponds to regions that were cooler and less dense.

One particularly interesting feature of the variations in the brightness of the CMB is that there is a characteristic scale to the fluctuations. Most of the bright and dark spots are around about one half of a degree across, so these spots would appear on the sky to be about the same size as the Sun or the full Moon (again, see figure 3). Since all of photons that make up the CMB have been traveling through space for the same amount of time since decoupling, these features must all be the same distance away and this unique angular scale must correspond to some particular physical length scale in the universe at decoupling. Although one could imagine a variety of exotic processes which could single out a particular length scale in the early universe, the statistical properties of the fluctuations in the CMB on all scales indicate that this length scale really corresponds to the **horizon scale at decoupling**.

The horizon scale at decoupling is simply the distance light could travel between the Big Bang and the end of decoupling, which is a calculable quantity. For most of the time between the Big Bang and decoupling, most of the energy density in

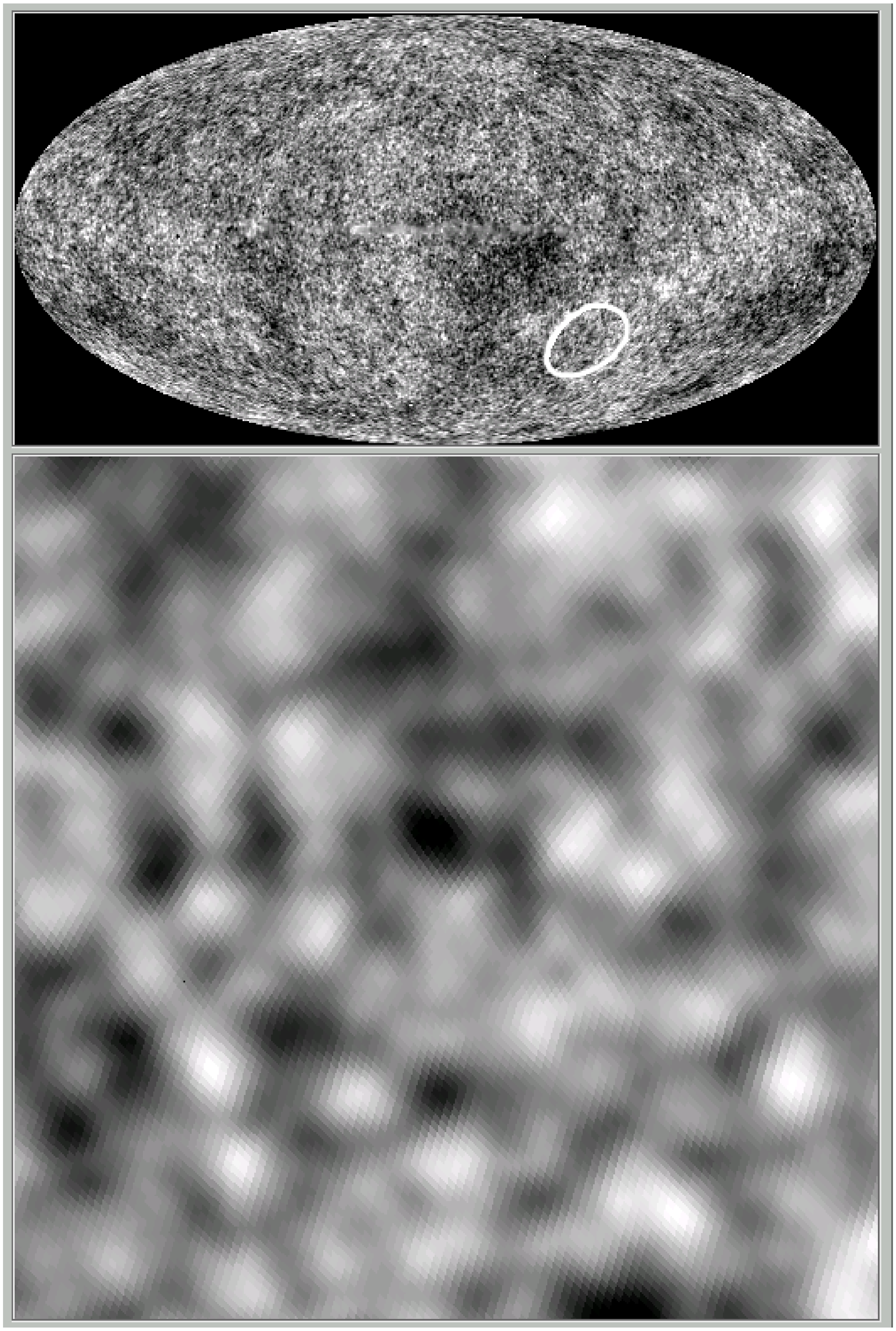
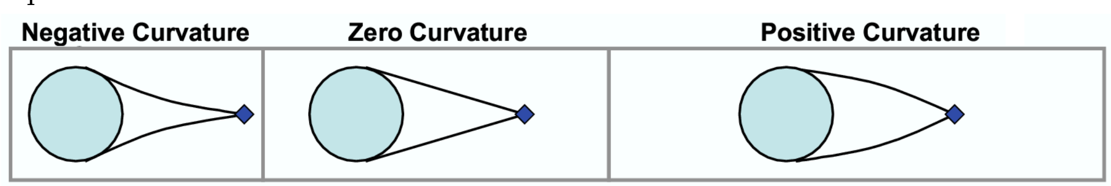


Figure 3: Variations in the brightness of the CMB measured by the WMAP satellite (this is the processed data from Mag Tegmark available at `http://www.hep.upenn.edu/~max/wmap3.html` ). The various shades of gray indicate variations in the brightness, with the whitest regions being 400 microkelvins (about one part in 10,000) brighter than the darkest regions. The top image depicts the entire sky, while the bottom image shows a close up of the circled region. Note that the bright and dark spots have a characteristic size, which is about half a degree on the sky.

the universe was stored in the photons themselves, so the amount of radiation in the universe determines the expansion rate. The amount of time it takes for the universe to expand enough for decoupling to occur therefore does not depend much on the amount of matter or Dark Energy of the universe, but instead depends on the number of photons in the universe, which we know based on the intensity of the CMB today. The age of the universe at decoupling can therefore be calculated reasonably reliably and it works out to be about 400,000 years. The horizon scale of decoupling is therefore 400,000 light years, and so the features observed in the CMB which appear to be about half a degree wide are actually roughly 400,000 light years across.

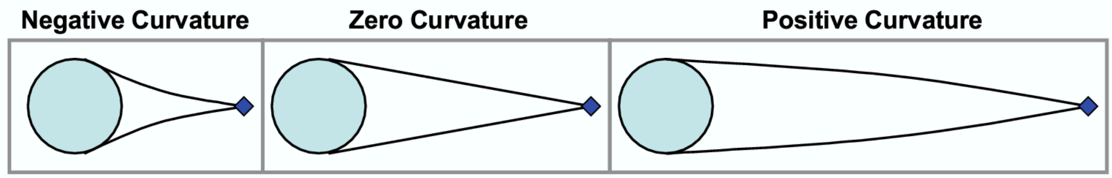
Having estimates of both the apparent and actual sizes of these primordial objects places important constraints on the global geometry and the total energy density of the universe. As mentioned above, the energy density of the universe can distort the geometry of space and time. In particular, it can induce **curvature** into the spatial geometry. If the curvature of the universe is zero, then spatial geometry follows the usual euclidean rules: parallel lines never intersect, the sum of the internal angles of a triangle is always 180°, and so on. However, this is a special case, which requires that the present energy density of the universe today has a particular value called the **critical density** (It turns out that if the universe has zero curvature at one time, then it will always have zero curvature, so the critical value of the density changes with the scale factor). If the energy density is higher than this, the universe must have positive curvature, if it has less, then it has negative curvature. In both these cases the geometry of the universe deviates from the usual euclidean rules. For example, if the curvature is positive, the sum of the internal angles of a triangle is always greater than 180° (as it is on the surface of sphere), while if the curvature is negative, these angles always total less than 180°.

These alterations to the spatial geometry affect the apparent sizes and distances of objects. For example, imagine there are three observers (diamonds) viewing the same sized object (circles) in universes with different geometries, as shown in the three panels below:




In the middle panel, the geometry has zero curvature, so light travels along the truly straight lines shown in the image. In the other two cases, the curvature of the universe causes the light to follow somewhat curved paths. The apparent size of the object as seen by the observer depends of the angle between the two lines which connect to the edges of the object. Say the angle between the two lines in the zero curvature case is 10°, then the object will appear 10° across to the observer. On the other hand, in the negative curvature case, the angle between the lines is smaller and the object will appear less than 10° across for this observer. Conversely, in the positive curvature case, the angle between the lines is larger and the object appears more than 10° across. Thus objects of the same size which are the same distance away from the observers have different apparent sizes due to the changing geometry of the universe.

Now imagine that the same sized object has the same angular extent in the three different cases. Then the angle between the two lines connecting the observer to the edges of the object must be the same. This was not the case when the observers were all the same distance from the object, so the observers must be different distances from the object in the three cases, as shown here:




Thus, given actual size of an object and the angular size of the object on the sky, the distance to the object depends on the curvature and total energy density of the universe. Similarly, in order for features 400,000 light years across at decoupling to appear half a degree wide today in the CMB, the distance traveled by the light between decoupling and today depends on the curvature of the universe. If the curvature is more positive, then the light has a longer distance to travel, and if the curvature is more negative, then the relevant distance is shorter.

The amount of time the light has to complete the journey also depends on the total energy density of the universe. Remember that wavelength of the photons that make up the CMB has increased by a factor of 1000 between decoupling (when there were just enough ultraviolet photons to support an ionized plasma) and today (when the photons have a typical wavelength of about a millimeter). The scale factor of the universe therefore increased by a factor of 1000 between decoupling and today. If the curvature of the universe is more positive, the energy density of the universe is higher and the expansion rate is faster, so it takes less time for the universe to expand by this amount. Conversely, if the curvature of the universe is more negative, the universe takes a longer time to expand by the required amount.

Notice that as the energy density of the universe increases and the curvature becomes more positive, the distance that the photons have to travel *increases* while the amount of time they have to complete the journey *decreases*. Since light travels at a known, finite speed, there is only one unique value for the curvature and the total energy density where the light can travel the required distance in the allowed time. Thus in order for the appropriate features in the CMB to have their observed size today the curvature of the universe must have a particular value, which turns out to be very close to zero. The total energy density of the universe must therefore be very close to the critical density.

## 3 The Age of the Universe

Given the total energy density of the universe from the CMB and the ratio of matter to dark energy from the supernova data, one can compute exactly how the scale factor of the universe should change with time and when it would have been effectively zero. With these constraints on the composition of the universe, the age of the universe can now be estimated to be around 13.6 billion years, with an uncertainty of only a few hundred million years.

In fact, the CMB data alone contains enough information to derive this estimate for the timing of the Big Bang. However, the data from the supernovae as well as other observations confirm that this calculation is consistent with all of the available information about the universe. Even so, it is quite possible that our understanding of how the universe operates and evolves with time is incorrect when the density of the universe was very high, so there is still the possibility that the universe existed in some form before the time when our calculations indicate that the scale factor was zero. Hopefully future observations of primordial gravity waves and other cosmological phenomena will be able to provide important information about times when the scale factor was extremely small and yield new insights into the origin of the universe.

## 4 Conclusions and Acknowledgments

With this estimate of the age of the universe, we come to the end of the 59th Compton Lecture Series. I would like to thank all the people who helped me with these lectures: First, thanks to Bruce Winstein and James Pilcher for encouraging me to do these talks and to take on such a multi-disciplinary topic. Also, thanks to the people in the EFI and KICP who helped with the practical matters of making these lectures happen, especially Nanci Carrothers, Charlene Neal and Dennis Gordon. I would also like to thank all the members of the CAPMAP collaboration, especially Dorothea Samtleben, who put up with me while I worked on these talks, and even came to a few lectures. I also want the thank the following people who provided and helped me understand some of the content of these lectures: The members of the Chicago Maya Society, K.E. Spence, John C. Whittaker, Wen-Hsiung Li, Robert Clayton, Stephen Simon, Andrey Kravtsov, James Truran, Stephan Meyer, Erin Sheldon and Wayne Hu. (Of course, any errors in these lectures are my responsibility alone). Also, thanks to all the people who came to the lectures. I hope you all enjoyed these talks and gained a new appreciation for the folks who attempt to make sense of the Age of Things

## 5 References

As with the previous two lectures, a good book for getting an overview of astronomy and cosmology is:

- Freedman and Kaufmann *Universe, Sixth Edition* (Freeman and Company, 2001)

However, cosmology has been advancing so rapidly and new data is appearing so quickly that no book is completely up-to-date. Review articles in various journals and magazines are the best way to keep up. Some reasonably accessible recent articles (with references) are:

- "Cosmology in the New Millennium" by W.L. Freedman and M.S. Turner in the October 2003 issue of *Sky and Telescope*
- "Four Keys to Cosmology" by various authors in the February 2004 issue of *Scientific American*

For more details on the WMAP satellite, see the special WMAP issue of the *Astrophysical Journal Supplement Series* Volume 148 (2003).