The Age of Things: Sticks Stones and the Universe
Eighth Lecture Notes
Matt Hedman
http://cfcp.uchicago.edu/~mmhedman/compton1.html
5/15/2004

Colors, Brightness and the Age of Stars

The sun and other stars are impressive objects. These giant balls of mostly hydrogen gas are so massive that nuclear fusion occurs in their cores, which releases copious amounts of energy. This energy heats the stars and causes them to shine so brightly that their light can be seen over great distances. This light contains clues about the inner workings of the stars, and can even provide estimates of their age in certain circumstances.

1 Studying starlight

The most obvious characteristic of a star in the sky is its brightness, which indicates how much light the star produces (provided, of course, that we know how far away the star is). The total amount of light produced by a star is measured in terms of a **luminosity**, or the total amount of energy carried in the light emitted by the star in a given time. For convenience, the luminosity of a star is usually reported relative to the luminosity of the Sun (For example, a star might be said to have ten times Solar Luminosity).

After brightness, color is the most basic characteristic of starlight. Some stars appear reddish, others yellow, yet others bluish-white. These colors reflect differences in how much light the stars emit at different wavelengths, which can be quantified by plotting a **spectrum** of the object's brightness as a function of wavelength. The spectra of stars have a characteristic shape, illustrated schematically in figure 1 with a broad peak that occurs at different positions for different stars. These shapes are typical of thermal radiation, which is produced by objects at a finite temperature due to random motions of their component atoms, etc. The shape of a thermal spectra is not sensitive to the composition or structure of the object, but instead depends mainly on its temperature. As the temperature of the object increases, the peak of the spectrum moves to shorter and shorter wavelengths. Thus a object with a bluish glow is hotter than one with a reddish glow. The colors of the starlight therefore provide a measure of the **temperature** of the stars.

In principle, both the luminosity and the temperature of the star could be extracted from a well calibrated spectrum. A complete spectrum gives the amount of light seen at every wavelength, so we can compute the total amount light received at all wavelengths and (given the distance to the star) infer its luminosity. Also, by fitting the shape of the spectrum to the correct form, we can determine the temperature of the star.

In practice, for large number of stars it is too time-consuming to obtain complete spectra, and simpler methods are used to trace the luminosity and temperature of the stars. These methods use a series of filters which select out one part of the spectrum. The various filters are identified by a letter that indicates which part of the spectrum they allow to pass into the detector. For example, the B filter transmits primarily

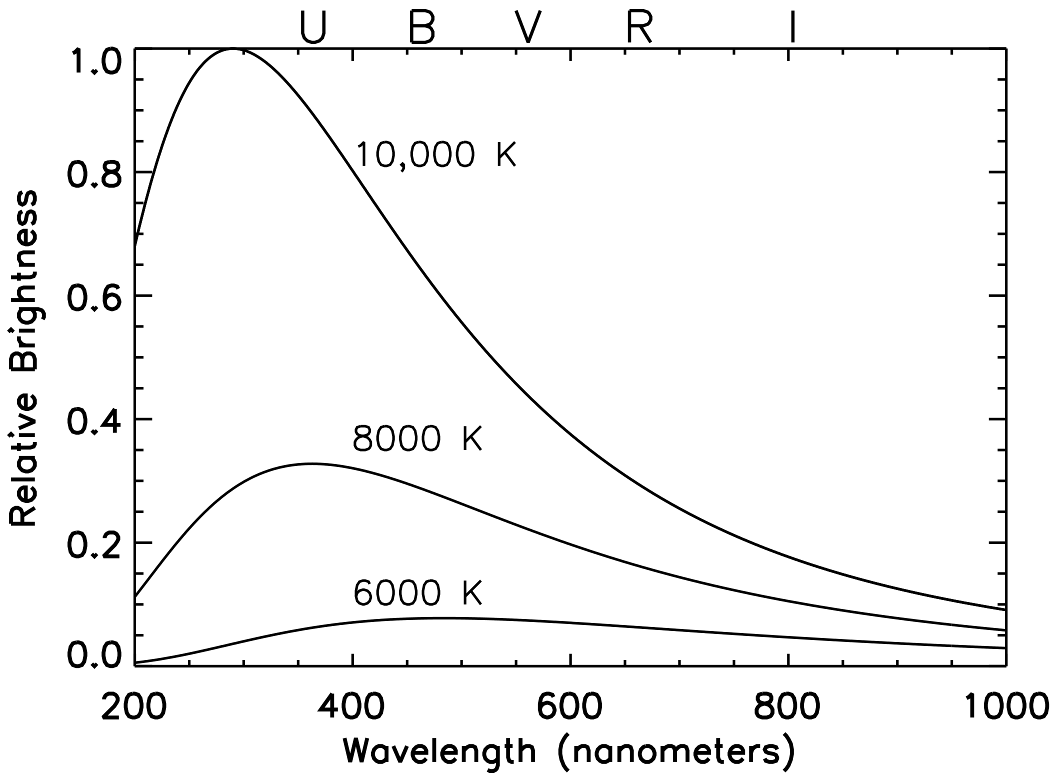

Figure 1: Schematic stellar spectra, which shows the brightness plotted as a function of the wavelength of the light (Blue is towards the left, red is towards the right). The actual spectra of stars have a number of dips and wiggles due to various elements in their atmospheres. However, the basic shape of the spectra is a broad hump, as shown here. This shape corresponds to the thermal emission of objects at different temperatures. The labels on each curve indicate the corresponding temperature for each spectrum. Note that higher temperature objects have a spectrum which is peaked at shorter wavelengths. The letters along the top of the plot represent some of the filters commonly used to measure starlight.

blue light, while the V filter transmits longer-wavelength *visible* light (see figure 1 for some of the other filters).

The total amount of starlight transmitted through the filter and detected is usually measured in terms of a **magnitude**. A star with magnitude 1 is about 2.5 times brighter than a star with magnitude 2, which is in turn 2.5 times brighter than a star with magnitude 3 (note that the greater the magnitude, the lower the brightness). The *difference* in the magnitudes of two stars therefore gives the *ratio* of their brightness.

The difference in magnitudes measured with two different filters depends on the shape of the spectra and therefore traces the temperature of the star. These differences are referred to as **colors** in astronomical parlance, and are denoted by a pair of letters, such as $B - V$. Realize that if a star has a *smaller* value of $B - V$ means it is *bluer* and *hotter*. This is because the star has more light in the blue and so has a smaller magnitude in B than in V.

The luminosity of the star is derived from the magnitude of the star measured with one particular filter, such as V. Clearly, the fraction of the light that passes through the filter depends on the color of the star, but such effects can be corrected for relatively easily (especially when the peak of the thermal spectrum is towards the blue or short-wavelength side of the filter).

A much more significant correction is required to account for variations in the distances to different stars. While we cannot go into all the details of how to measure the distance to astronomical objects here (this will be an important part of the next lecture), we can at least note that the locations of nearby stars can be measured using **parallax**. Parallax is the apparent change in the position of the object due to a change in the position of the observer, which depends on how far the object is from the observer. For example, imagine you are on a train watching the scenery go by, the nearby trees move across your field of view quickly, while the mountains in the distance appear to move much more slowly. Similarly, as the earth moves around the sun, the stars appear to move around by tiny amounts. By carefully measuring the apparent positions of the stars at different points during the year, it is a straightforward exercise in geometry to estimate how far away it is.

Once the distance to the star is known, the **apparent magnitude** of the star can be converted into an **absolute magnitude**, or the magnitude the star would have if it were located some fiducial distance (10 parsecs or about 30 light years) away. The absolute magnitude of a star then depends only the amount of light it emits, so it is straightforward to calculate the luminosity from this parameter (for reference, the absolute magnitude of the sun is about -4.8).

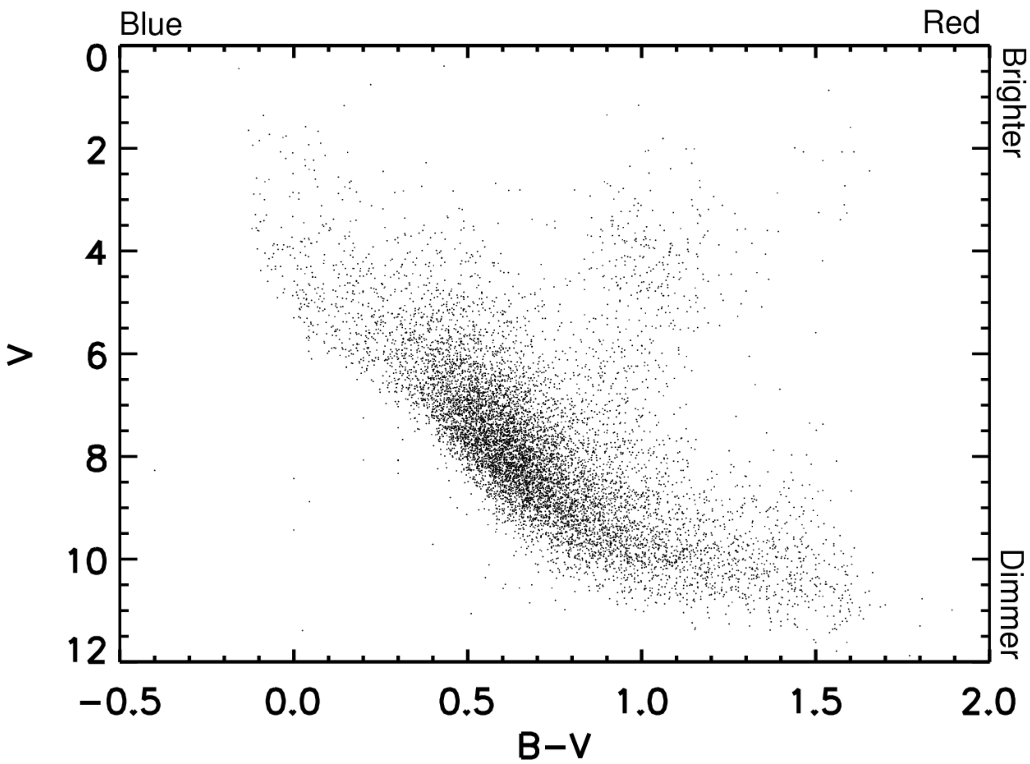

Figure 2: A Color-Magnitude Diagram of nearby stars, based on data from the Hipparcos Satellite available at `vizier.cfa.harvard.edu/vizier`. This is a plot of V Absolute magnitude versus the color $B - V$, and each point represents a single star. Points that lie towards the left of the plot are bluer than stars that lie to the right of the plot, and stars towards the top of the plot are more luminous that stars towards the bottom of the plot. Most of the stars fall along a diagonal line known as the main sequence

2 The Main Sequence

Given a number of stars with known distances, we can make a plot of absolute magnitude versus color (in other words, luminosity versus temperature). This plot is called a **Color-Magnitude Diagram**, (also called a Hertzsprung-Russel Diagram), and an example for nearby stars is shown in figure 2. Most of the stars fall along a diagonal line across the plot, which is known as the **main sequence**. For these stars, there is a correlation between temperature and luminosity. Hotter (bluer) stars are more luminous than colder (redder) stars. Also, there is a spray of **red giant** stars in the upper left hand corner of the plot.

Similar plots can be constructed for different groups of stars, such as the stars in the so-called **globular clusters**. These are roughly spherical masses of up to a million of stars crowded very tightly together. The color-magnitude diagram of the stars in such a cluster (shown in figure 3) looks quite a bit different than the previous one. The stars seem to fall into clumps and lines that snake around in different directions. Note in particular the diagonal line towards the bottom of the plot. This corresponds to the main sequence stars in the previous figure, but the bright, blue end of the sequence is missing. There are bright-blue stars in the cluster, but they fall along

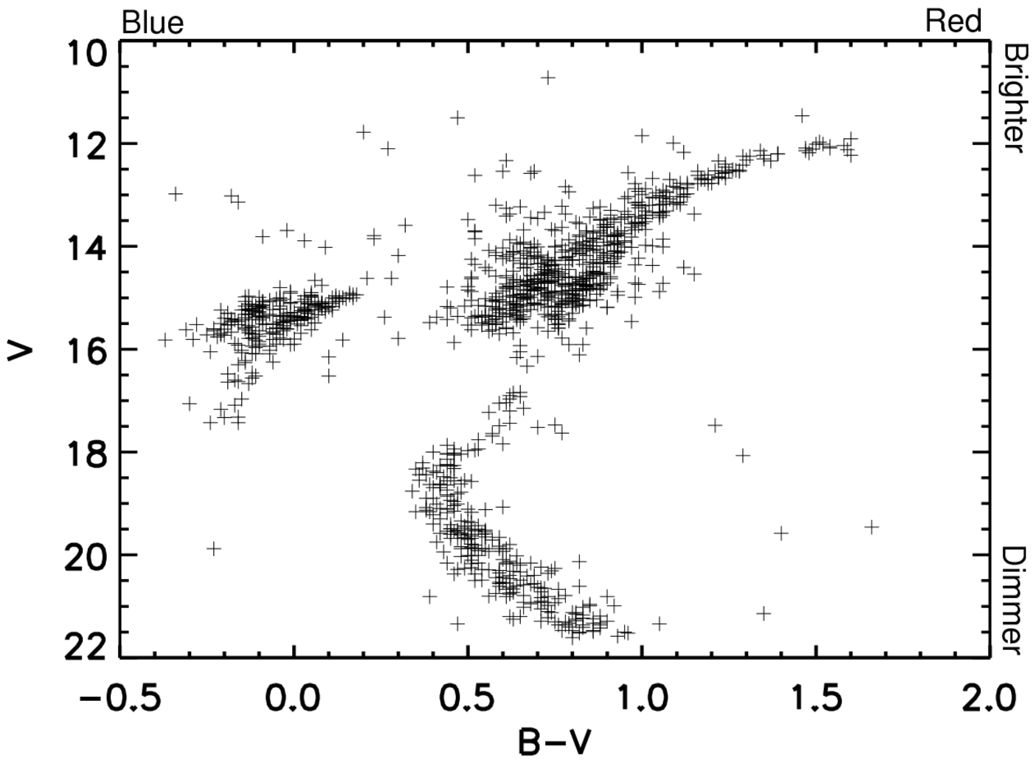

Figure 3: Color-Magnitude diagram of a globular cluster M13 (from data available at `vizier.cfa.harvard.edu/vizier`). This diagram should be compared with the diagram in figure 2. Note in particular that the diagonal line corresponding to the main sequence near the bottom of the plot appears to be cut off on the blue end. (The values of V on the y-axis are larger here than in the previous figure because this plot shows apparent magnitudes instead of absolute magnitudes. Since all these stars are the same distance away, this just offsets the magnitudes of all of the stars by the same amount.)

a **horizontal branch** instead of the diagonal line of main sequence stars. Different clusters show the same basic features, but the point at which the main-sequence ends is different for different clusters. It turns out the location of this **main-sequence turn-off** provides a way to estimate how long ago the stars in the cluster formed.

3 The Lives of Main Sequence Stars

3.1 Nuclear Reactions

Main sequence stars exist in a almost perfect state of equilibrium where the pull of gravity is balanced by an outward push generated by the fusion of hydrogen nuclei. All stars (indeed, the entire universe) are composed primarily of hydrogen, which is the simplest element, with a nucleus typically consisting of a single proton. In the core of a main-sequence star, these protons are assembled into helium nuclei, made up of two protons and two neutrons.

Figure 4 shows one of the methods by which four protons become one helium nuclei (other processes can occur if heavier nuclei exist to catalyze various transformations).

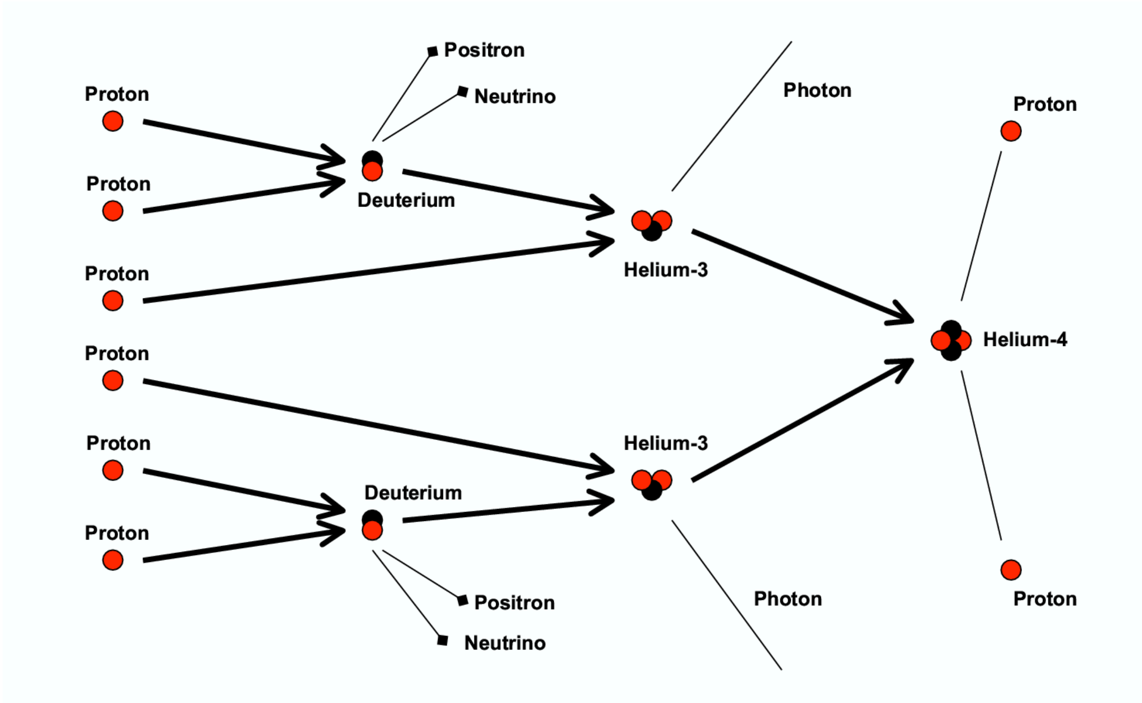

Figure 4: The fusion of hydrogen into helium. Hydrogen nuclei (individual protons) can be assembled into a Helium nuclei is a variety of ways. the process illustrated here is the most straightforward (other process use heavier nuclei as catalysts) Each of the reactions illustrated here reduces the mass of the system and so releases energy into the motion of the various particles

First, two protons form a nucleus of deuterium, a heavy form of hydrogen with one proton and one neutron (since one proton converts into a neutron during this process, a positron, or anti-electron is emitted during this process). Next, another proton combines with this deuterium nucleus to form a light variant of the helium nucleus with two protons and one neutron (a photon is emitted during this process to conserve momentum). Finally, two of these light helium nuclei come together and two protons are thrown off, leaving behind a normal helium nucleus, with two protons and two neutrons.

After each of these reactions, the mass of the system the reaction is slightly lower than it was before. Recall from the third lecture that the famous relation $E = mc^2$ says that there is an energy associated with any mass. Therefore, if the mass of a system is reduced, there is energy available which goes into the relative motions of the various particles. The total amount of energy released can be computed from the roughly 1% mass difference between four protons and a helium nucleus. This number is quite large, since converting one gram of hydrogen atoms into helium releases as much energy as burning 20 metric tons of coal.

Even though fusion releases large amounts of energy, we do not observe this process happening in our day to day lives, and indeed it is difficult to get it to happen in any laboratory. This is because protons are all positively charged, so they repel each other, which usually prevents them from getting close to each other. Only if these nuclei are pushed together with enough force to overcome this repulsion will they can get close enough that the nuclear forces become dominant and the fusion reaction can proceed. Fusion will therefore only occur if the pressure and temperature is sufficiently high, such as in the core of the star.

Note that even if we have an environment where hydrogen nuclei can be assembled into helium, we cannot just continue to assemble larger and larger nuclei. This is because beyond Helium-4, adding more protons and neutrons generally causes the mass of the system to increase instead of decrease. Therefore these reactions require an energy to be poured into the system from the outside (perhaps from the energy released from other reactions) so they occur rarely. Those reactions which do result in a mass decrease (such as a Helium-3 and Helium-4 nuclei combining to form a Lithium nucleus) eventually result in a nucleus that falls apart into two Helium-4 nuclei.

It turns out that the reaction that mostly efficiently creates nuclei larger than helium and releases energy is the fusion of three Helium-4 nuclei into Carbon-12. However, this process requires three helium nuclei to be very close to each other at the same time, and thus requires much higher densities and pressures than is required to convert hydrogen into helium, and these conditions are not found in the core of a main-sequence star. Therefore, the fusion of hydrogen to helium is practically the only source of energy released from the core of a main sequence star.

3.2 Balancing Gravity

The fusion of hydrogen keeps the star from collapsing under its own gravity. The energy released from these nuclear reactions goes into radiation and the motion of particles, which carry a flow of momentum out of the core that counteracts the inward flow of material due to gravity. Fusion energy exactly balances gravity in main sequence stars because as the pressure on the core increases, the nuclei are pushed closer together and the rate of nuclear reactions increase. Thus if the rate of fusion was not enough to support the star, it would begin to collapse, increasing the pressure on the core. The rate of nuclear reactions would then increase until it stopped the collapse of the object. Conversely, if the energy released by fusion reactions were more than what was needed to support the star, material would be blown outward, reducing the pressure on the core. The nuclear reaction rate would then drop until a state of equilibrium was reached.

Note that while the star is in equilibrium, there is no net flow of material inward or outward. Any particle spends as much time falling to the center of the star under gravity as it spends flying outward because it just collided with some particle moving away from the core. This means that outside the core the star is neither gaining or losing energy, so energy must be escaping the star in the form of light and other radiation as quickly as it is being generated in the core. Therefore the luminosity of the star is directly related to the rate of nuclear reactions in the core.

3.3 Mass and Luminosity

The luminosity is not only a measure of the rate of nuclear reactions of the core of the star, but also provides a means to estimate the mass of the star. The equilibrium energy production rate in the core must support the weight of the star, it should make sense the the fusion rate and the mass of the star are correlated with each other. Imagine we have two stars, A and B, where the mass of star A is twice that of

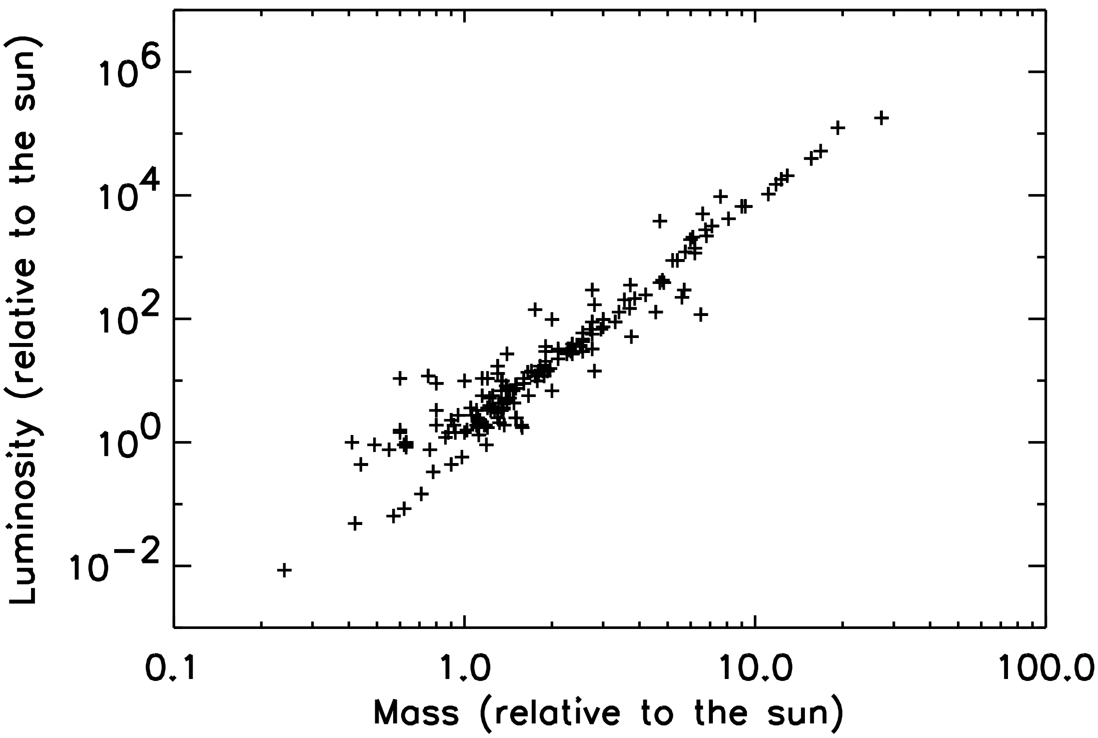

Figure 5: The Mass-Luminosity relation (Based on data from Svetchnikov and Bessonova available at `vizier.cfa.harvard.edu/vizier`). This is a plot the luminosity of the star versus their mass. Note that when the mass increases by 10, the luminosity increases by several thousand.

star B. Star A therefore has twice as many particles to support, and the gravitational pull on each particle is twice as strong, so four times as much energy is needed to keep the material aloft in star A. Furthermore, the particles in star A must move more quickly to keep from collapsing into the center of the star, so they collide more frequently and energy is carried out to the surface of the star more quickly. Thus energy escapes from the surface of star A more rapidly than star B and more energy must be supplied by the core to compensate. All in all, star A must have a nuclear reaction rate (and luminosity) roughly ten times higher than star B to support its excess mass.

This relationship between mass and luminosity in main sequence stars can actually be observed and measured. We have already discussed that the luminosity of a star can be measured if its distance from us is known. However, the mass of a star can only be directly measured if it has a companion in orbit around it, as in a binary star systems. Careful observations of how stars in binary systems move with respect to each other over the years has yielded mass estimates for a fair number of main-sequence stars. If we plot the luminosity of these stars as a function of their mass, as shown in figure 5, then we indeed see there is a relatively tight correlation between mass and luminosity, and a relatively small change in mass corresponds to a much larger change in luminosity. This not only indicates that we have a reasonable model for how these stars work, but also provides a basis for estimating how long they live on the main-sequence.

4 The End of Main Sequence Stars

Main sequence stars can only exist in a state of equilibrium as long as they are able to assemble hydrogen atoms into helium. Eventually the amount of nuclear fuel in the core of the star runs out and the star falls out of equilibrium. Clearly the star cannot maintain its equilibrium state past the time when it would convert all of its hydrogen to helium, so there is a clear upper limit to how low a main-sequence star can last.

In reality, a main sequence star never lasts long enough to convert all of its helium into hydrogen. This is because helium is heavier than hydrogen and so it accumulates in the core. Helium nuclei cannot undergo fusion in the core of a main sequence star, so the only thing keeping them from collapsing is the energy released by the fusion of hydrogen around them. These reactions are able to support the full mass of the hydrogen-rich star, but they can only support a fraction of this mass in a helium-rich core. Helium has four times the mass of Hydrogen, so it requires more energy and must move more quickly to keep it from falling into the center of the star. It turns out that hydrogen fusion can only support a helium-rich core so long as it is less than about 10% of the total mass of the star (this is the so-called **Schonberg-Chandrasekhar Limit**). Depending on the size of the star, there are different physical processes inside the star which become important as this limit is reached, but the general result is the same: The star loses its ability to maintain equilibrium when about 10% of the mass of the star in converted from hydrogen to helium. When this happens, the core of the star collapses and the outer layers expand. This reduces the apparent temperature of the surface and the increases the luminosity of the star, so it moves away from the main-sequence towards the upper left hand corner of the color-magnitude diagram to become a red giant.

The luminosity of the star is a measure of how fast hydrogen is being converted into helium in the core of the star. Also, using the mass-luminosity relation, we can estimate the mass of the star. We can therefore calculate how long it will take for the star to convert roughly 10% of its mass to helium, and thus how long the star will last before it turns into a red giant.

For those readers who are interested in a mathematical example, consider our sun. The luminosity or total energy output of the sun is $4 * 10^{26}W$, which means about 4 billion ($4 * 10^9$) kilograms of mass are being converted into motional energy energy second. About 1% of the mass is converted into energy when hydrogen fuses into helium, so this means 400 billion ($4 * 10^{11}$) kilograms of hydrogen is being converted into helium every second. This means about $1.4 * 10^{19}$ kilograms of hydrogen is converted to helium every year. The mass of the sun is about $2 * 10^{30}$ kilograms, so we expect that it will take the sun about 14 billion years for the sun to convert 10% of its mass to Helium and become a red giant. Doing more detailed calculations show the sun should live about 10 billion years, so this back-of-the-envelope calculate is not too far off.

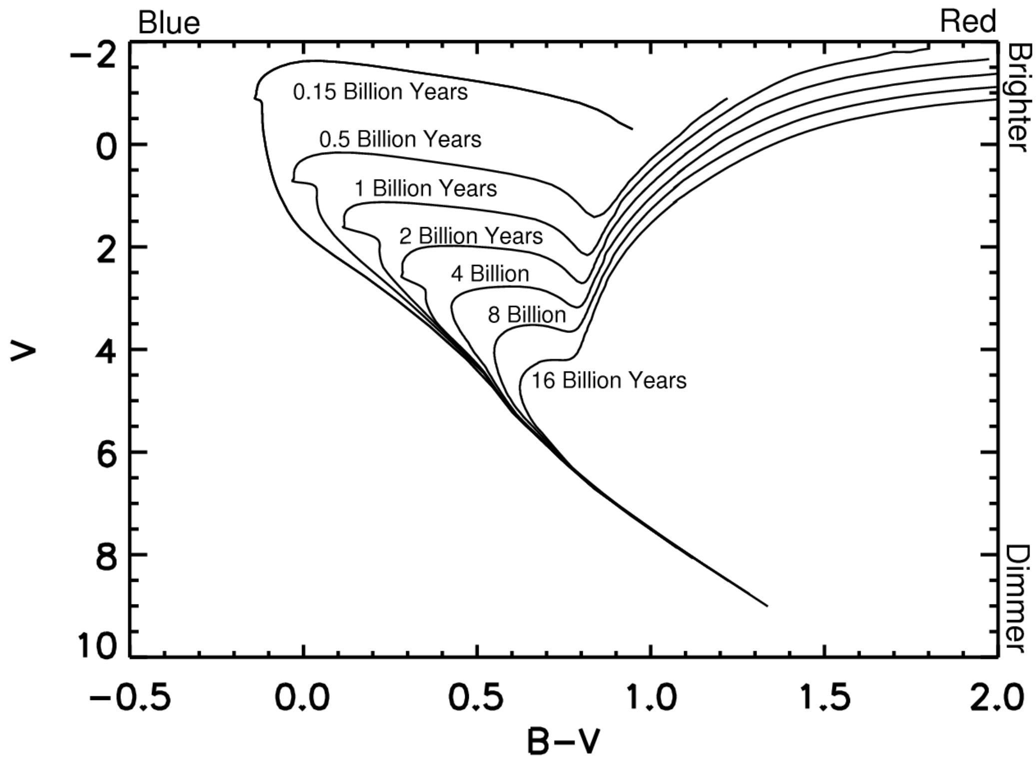

Figure 6: Schematic color-magnitude diagrams of a collection of stars all created at the same time, at different times after their creation. First, the brightest bluest stars run out of hydrogen first and become red giants, but as time goes on, redder and dimmer stars leave the main sequence.

5 The Changing Main Sequence

Although we can estimate how long a given main sequence star will live based on its luminosity, we cannot easily use this calculation to estimate how long ago an individual star formed. This is because while in equilibrium these stars do not change in appearance very much, so it difficult to tell if the star formed relatively recently or several billion years ago. However, if we have a collection of main sequence stars formed at the same time, then an age for this collection can be estimated because the different stars have different lifetimes.

Recall that the mass-luminosity relation shows that if two stars differ in mass by factor of two, then they differ in luminosity by a factor of about 10. This means the rate of fusion in the core is a factor of 10 higher in the more massive star. However, this star only has twice as many hydrogen atoms. Therefore it will run out of fuel five times faster than the lower mass star. The bright blue main sequence stars therefore have much shorter lifetimes than the dimmer, redder stars.

Imagine a collection of main sequence stars were all created at the same time. At this time, we would have a complete main sequence. However, before long the brightest, bluest stars will run out of fuel and convert into red giants. Thus stars near the tip of the main sequence move off to the right (red) side of the color-magnitude diagram. As time goes on, progressively dimmer and redder stars move off the main

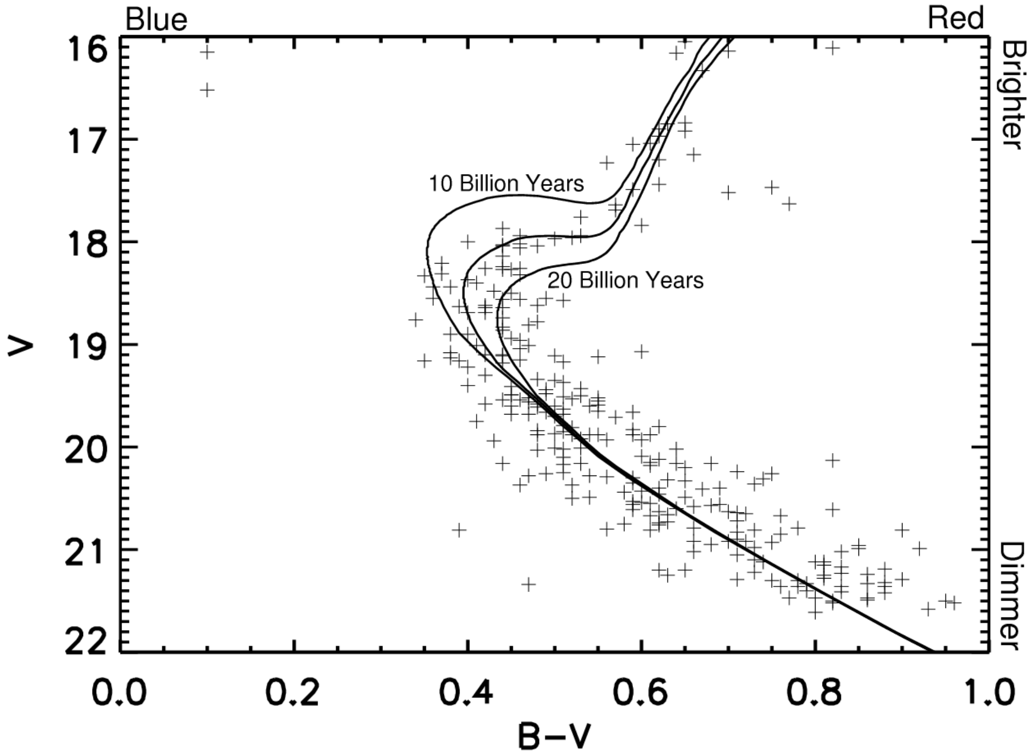

Figure 7: A Closer look at the main sequence part of M13 illustrated in figure 3, with the theoretical curves for 10, 15 and 20 million year old clusters. This illustrates how the shape of the main sequence turn off can indicate the age of the cluster, and the challenge involved in measuring ages with an accuracy of a billion years

sequence. The color-magnitude diagram of the stars therefore changes with time is a reasonably calculable way, as shown in figure 6. The brightest, bluest stars left on the main sequence at any time are just about to become red giants. Therefore, the amount of time that has elapsed since the collection of stars formed is the same as the calculable lifetime of these brightest stars

Much of the color-magnitude diagram of globular clusters resembles these theoretical curves. It is therefore reasonable to expect that these clusters consist of a collection of stars created around the same time. By comparing these data to different theoretical curves, we can estimate when the stars in the cluster formed.

Figure 7 illustrates both the promise and the difficulties of this method of measuring age. It shows a blow of the main sequence data with three different theoretical curves (moved around so they more-or-less match "by eye" for purposes of illustration). Clearly the data more closely follow the 15 billion year old curve then either the 10 or 20 billion year curves. However, we cannot be more precise than this by just looking at the data.

In practice, much more sophisticated techniques (and a lot more stars) are needed to make a precise measure of age from the shape and position of the main-sequence turn-off. Also, there are other factors besides the mass/luminosity of the star (such as the amount of helium and other elements in the star originally) that affect how fast it evolves any from the main sequence and that must be taken into account to obtain accurate ages.

Recent estimates of the age of the globular clusters have been made that attempt to take into account these various issues. These analyses find that the the old globular clusters are about 12 billion years old, with an uncertainty of about a billion years. Now other methods of estimating the age of these objects do exist (some utilize characteristics of the white dwarfs present in the cluster, other employ the amounts of detectable radioactive nuclei in the stars), which generally agree with these estimates. Globular clusters therefore appear to be very ancient objects, and indeed they place important constraints on the age and early evolution of the universe, which is the subject the next two lectures.

6 References

For a good general introduction to astronomy, see:

- Freedman and Kuafmann *Universe, 6th Ed* (Freeman and Co, 2001)

Progressively more detailed works on stellar astronomy can be found in

- R.J. Taylor *The Stars: their structure and evolution* (Cambridge 1994)
- R. Kippenhahn and A Weigart *Stellar Structure and Evolution* (Springer-Verlag 1994)

A good recent review of the issues involved in dating Globular Clusters is

- B.W. Carney and W.E. Harris *Star Clusters* (Springer 2000)

Recent articles on main-sequence turn-off dates are:

- B. Chaboyer "The age of the Universe" in *Physics Reports*, Vol 307 (1998) pp 23-30
- R. Gratton et. al. "Age of Globular Clusters...." in *Astrophysical Journal* Vol 491 (1997) pp 749.
- R. Jiminez "Towards an accurate determination of the age of the Universe" in *Dark Matter in Astrophysics and Particle Physics*

For other methods of measuring the age of Globular clusters, try:

- Hansen et. al. "White Dwarf cooling sequence of the Globular Cluster Messier 4" available at `www.arxiv.org astro-ph/0205087`
- Truran et. al. "Probing the Neutron-Capture Nucleosynthesis History of galactic Matter" in *Publications of the Astronomical Society of the Pacific*— Vol 114 (2002) pp 1293.